T.C.
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SOSYAL BİLİMLER ENSTİTÜSÜ
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EVIDENCE ON THE EXISTENCE OF NONLINEARITY AND CHAOTIC BEHAVIOUR ON STOCK EXCHANGE MARKETS:

“AN EVALUATION WITHIN THE CONTEXT OF TURKEY AS AN EMERGING CAPITAL MARKET”

Yüksek Lisans Tezi

IŞIL TOPÇUOĞLU

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Danışmanı: Prof. DR. ALİ OSMAN GÜRBÜZ

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Marmara Üniversitesi
Sosyal Bilimler Enstitüsü Müdürlüğü

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LIST OF ABBREVIATIONS

ISE	Istanbul Stock Exchange
EMH	Efficient Market Hypothesis
H	Hurst Exponent
BDS	Brock-Dechert-Scheinkman Test
CRSP	Center for Research in Security Prices
NEGM	Nychka-Ellner-Gallant-McCaffrey
CAPM	Capital Asset Pricing Model
i.i.d	Independent, Identically Distributed
MPT	Modern Portfolio Theory
R/S	Rescaled Range
ARMA	Autoregressive Moving Average Modeling
SDIC	Sensitive Dependence on Initial Conditions
LE	Lyapunov Exponent
ADF	Augmented-Dickey Fuller

SYMBOLS

$\mathbf{S_n}$ Standard Deviation of Sample

$\mathbf{R_n}$ Adjusted range

w BDS Test Statistic

$\boldsymbol{\lambda}$ Lyapunov Exponent

1. INTRODUCTION

Interest in understanding how capital markets have processed is always on the agenda of financial theorists. Many financial market models have been created to explain how capital flows from one investor to another, how the price of an asset has formed on the market and how stock indexes are rising and falling. Theorists have tried to understand the complex structure of financial markets as a dynamical system with the relationships between the many interdependent variables which change over time. Chaos theory, which is the study of complex nonlinear dynamical systems, is a new tool made available to researchers looking for deterministic patterns in apparently random series of data like the time series data of a stock market index (Peters, 1994).

Many of the models commonly used in empirical finance are linear regression models however many aspects of economic behavior in real life may not be linear and can not be predicted with an equation with two variables and being purified from other effects in the system (Peters, 1991). For example, for security's returns to be forecast in a statistically meaningful manner the returns must necessarily have a correlation with some variables over time. These variables may be past realizations of the security's own returns, or potential factors such as dividend yields, the earnings-price-ratio, or trading volume. However, this dependence violates the conditions of the simple random walk model of stock price behavior generally accepted in finance literature.

Recently algorithms have been proposed to try to distinguish between data generated by a "deterministic" and a data generated by a "random-stochastic" system. Dynamic systems can be "deterministic" or "stochastic" (Ajit, 2003, p.21-22). Chaos which is the study of complex non linear dynamical systems generally shows stochastic manner means that there are many possible solutions with a probability of occurrence (Ajit, 2003, p.22). However, the type of chaotic behavior we have mentioned above is deterministic. In deterministic form, there is only one solution for every state means that precise knowledge of the conditions of the system at one time allows us, at least in principle, to predict exactly the future behavior of that system. Therefore, "chaos theory in finance" has tried to reconcile the stochastic form-randomness, which is also the evidence of market efficiency- and determinism-which is an evidence against market efficiency, with the other words chaos theory has searched for order in disorder (Ajit, 2003, p.23). Chaos theory has also advocated that although chaotic systems are deterministic they are unpredictable, yet. This is due to the feature of chaotic systems which is sensitive dependence on initial conditions. One consequence of this sensitivity is that seemingly insignificant adjustments to a system will be compounded over a time and can dramatically change the overall behavior of a system (Peters, 1994).

Over the past decades, numerous studies have documented the existence of nonlinear dependence and chaos in financial markets as an alternative to the linear models which enables analysis of very complex relationships with no simplifying assumptions. Such findings have important implications throughout the field of finance, both in academics and in applied finance (Ammermann & Patterson, 2003, p.176). For finance practitioners, the existence of nonlinear serial dependence raises the possibility of predictability in financial returns, even in the absence of return autocorrelation, or, alternatively, an exposure to greater levels of risk than would be expected under a random walk. For finance and econometricians, these findings raise concerns about the statistical adequacy of statistical models used to examine financial time series; for a specific application involving the weak-form efficient markets hypothesis.

Chapter Two will provide a review of the literature on nonlinearity and chaos with related empirical topics while *Chapter Three* will provide explanation for why classical financial models are not appropriate to forecast and accurately describe financial markets by mentioning inconsistencies of linear modeling, especially by comparing Capital Asset Pricing Model and Modern Portfolio Theory that have based on Efficient Market Hypothesis. In *Chapter Four,* technical concepts related to nonlinearity and chaos has been described and statistical tests to detect the nonlinearity and chaos has been introduced. In *Chapter Five*, the data used in the test of nonlinearity and chaotic behavior will be examined and test results will be summarized with an evaluation of the conclusions of these tests. Finally, *Chapter Six* will discuss the conclusions obtained from the previous chapters.

The study of chaos has proved new conceptual and theoretical tools enabling us to categorize and understand complex behavior that has confounded previous theories.

In the past few years, a large literature has appeared on non-linearity in finance and economics and these studies especially concentrated on testing economic and financial time series for the presence of nonlinear dependencies using various measures indicative of complex dynamics.

2. LITERATURE REVIEW

Hsieh (1991, p.2) has asserted the interest in a topic related to chaos and nonlinear dynamics in his article called "Chaos and nonlinear dynamics: Application to Financial Markets" as:

"After the stock market crash of October 19, 1987, interest in nonlinear dynamics, especially deterministic chaotic dynamics, has increased in both the financial press and the academic literature. This has come about because the frequency of large moves in stock markets is greater than would be expected under a normal distribution. There are a number of possible explanations. A popular one is that the stock market is governed by chaotic dynamics."

However, chaos and nonlinear dynamics has been around since the early 1960's with the idea of sequence of stock returns is a "fractal time series" which is introduced by Benoit Mandelbrot.

The objective of many nonlinear time series articles in finance has been to either fit a nonlinear model for stock return behavior, propose a new statistic to identify nonlinear behavior or test nonlinear behavior in time series and arrive at a yes/no response for whether or not nonlinearity exists (Skaradzinski, 2003, p.18).

The studies mentioned below sample the data at a variety of frequencies such as monthly, weekly, daily and intra-day. The first part of this review summarizes the idea of fractal time series which is proposed by Mandelbrot, second part summarizes the results of empirical research of nonlinearity and chaos on stock returns and stock exchanges and the third part outlines nonlinearity and chaos on other types of financial assets.

2.1. "FRACTAL" DYNAMICS BY MANDELBROT

Benoit Mandelbrot has contrasted white noise process with chaotic dynamical process for the sequence of increments in a time series. White noise process refers to a sequence whose increments are independently and identically distributed with zero mean and finite variance which reveals a normal distribution (synonymous with Brownian motion) in time series. During 1960's Mandelbrot believed that securities return did in fact follow a fractal time series and Brownian motion was not an adequate statistical description of the true process generating securities return. A fractal time series is one which is statistically "self similar" regardless of the time frame over which the increments of the series are observed. In order to resolve this inadequacy, Mandelbrot worked in two directions (Mandelbrot & Hudson, 2005, p.223-225). One direction involved relaxing the assumption of finite variance, which introduces what Mandelbrot termed the "Noah Effect". The other direction entailed relaxing the independence assumption termed as "Joseph Effect".

The Joseph Effect denotes the property of certain time series to exhibit persistent behavior more frequently than would be expected if the series were completely random, but without exhibiting any significant short-term dependence. To describe such processes, Mandelbrot broadened the idea of Brownian motion into the class of stochastic processes called "fractional" Brownian motion. Fractional Brownian motions exhibit complex, though linear, long-term dependencies and are characterized by a parameter called the Hurst exponent (H), which denotes the level of long-range dependence in the data and generally ranges from 0 to 1. If $H=0.5$, then the fractional Brownian motion will have no long-term persistence, and the result is standard Brownian motion, or white noise. If $0<H<0.5$, then the series will exhibit anti-persistence, as evidenced by a greater number of reversals and fewer and shorter trends than in a white noise series. On the other hand, if $0.5<H<1$, then the series will exhibit persistence, with fewer reversals and longer trends than the increments of Brownian motion. In this case, the graph would appear smoother than that of a random walk. In order to measure what sort of noise a time series most closely resembles, Mandelbrot developed a statistical technique called "rescaled range" analysis, which yields a measure of the Hurst exponent which will be examined in detail in further sections. Peters (1991, p.85) estimates the Hurst exponent to be 0.778 for monthly returns on the S&P 500 from January 1950 to July 1988. For a sample of individual stocks, Peters found (1991, p.87) Hurst exponents ranging from 0.75 for Apple Computer down to 0.54 for Consolidated Edison. All of these values are greater than 0.5, indicating a greater persistence among stock returns than would be expected if stock prices followed a geometric Brownian motion process.

The Noah Effect on the other hand, refers to the tendency of various time series with presumably independent increments, especially speculative time series, to exhibit abrupt and discontinuous changes. The existence of abrupt discontinuities in financial time series, combined with the empirical observation of both sample leptokurtosis* and unstable sample variances, led Mandelbrot to develop the "stable Paretian hypothesis". This hypothesis makes two basic assertions which are "the variances of the empirical distributions behave as if they were infinite" and "a stable paretian distribution is a member of the class of distributions that are "invariant" under addition. Stable Paretian distributions have four parameters α, β, δ and γ (Fama, 1963, p.422). $\boldsymbol{\alpha}$ is the characteristic exponent, β is the skewness parameter, γ is the scale parameter and δ is the location parameter. Under this hypothesis, stock returns follow a stable Paretian distribution with a characteristic exponent (α) whose value is between one and two (Fama, 1963, p.425). The characteristic exponent (α) determines the height of the extreme tails of distribution. When $\alpha=2$ the relevant stable Paretian distribution is the normal distribution (Fama, 1963, p.425).

Using the log price differences of daily cotton prices over various time frames as an example of a speculative time series, Mandelbrot (1963, p.394) uses basically visual techniques to make a number of interesting observations. First, "the empirical distributions of price changes are usually too 'peaked' to be relative to samples from Gaussian populations (Mandelbrot, 1963, p.394). Furthermore, while "the histograms of price changes are indeed unimodal and their central 'bells' remind one of the 'Gaussian

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* Leptokurtic distribution has a more acute "peak" around the mean-that is higher probability than a normally distributed variable of values near the mean- and "fat tails" –that is a higher probability than a normally distributed variable of extreme values.

ogive', there are typically so many 'outliers' that ogives fitted to the mean square of price changes are much lower and flatter than the distribution of the data themselves(Mandelbrot, 1963, p.395). The tails of the distributions of price changes are in fact so extraordinarily long. These observations are consistent with the 'stable Paretian hypothesis' Mandelbrot proposes. Via the use of graphs displaying the empirical probabilities of observations in the tail areas of the distributions, Mandelbrot obtains a value for α of 1.7, while the skewness and location parameters are estimated to be zero. Comparing the actual data to the probabilities obtained from a stable Paretian distribution with these parameter values, Mandelbrot finds that they produce a fairly good fit, although the actual data do seem to exhibit a slight amount of negative (left) skewness. However, the data still exhibit one major anomaly which the stable Paretian hypothesis cannot directly explain—"large changes tend to be followed by large changes—of either sign—and small changes tend to be followed by small changes[*]".

2.2. EMRIRICAL RESEARCH ON STOCK RETURNS AND STOCK EXCHANGES

Brockett, Hinich and Patterson (1988) tired to present statistical techniques for determining which time series are actually linear processes and which time series are not suitable for linear time series modeling. They applied the bispectral Gaussianity and linearity test to 10 different common stock daily time series to test whether a sample of a time series is consistent with the hypothesis that the observations are generated by a linear process. The results of the tests are daily returns are realizations of a nonlinear random process and there is a much higher degree of dependence in daily stock returns.

Scheinkman and LeBaron (1989) modeled weekly and daily returns (N=5200 daily returns) of the CRSP (Center for Research in Security Prices) value-weighted index and tested with the correlation dimension and the BDS statistic. They also investigated the behavior of individual stock return of Abbott Laboratories. The results point towards the presence of nonlinearity.

[*] The stock returns are characterized by volatility clustering where large returns are followed by large returns and small returns tend to be followed by small returns, leading to contiguous periods of volatility and stability.

Hseih (1991) examines the returns of the Standard & Poors 500 stock index (without dividends) for the following time periods: weekly returns from 1962 to 1989, daily returns from 1983 to 1989 and 15 minutes returns during 1988 divided into 4 approximately equal sub-samples. He finds that the weekly, daily and 15 minutes returns are not independence and identically distributed with the BDS test and concluded that the rejection of i.i.d is certainly consisted with the hypothesis that the stock market is governed by low complexity chaotic dynamics.

Abhyankar A., Copeland and Wong (1995) tested the presence of nonlinear dependence and chaos in real-time returns on the U.K.FTSE-100 Index, using a six month sample of about 60,000 minute-by-minute real time returns in the year of 1993. They used three nonlinearity tests which are Hinich Bispectral linearity test and BDS; then to test for chaos they used neural nets approach of Nychka et al. and Lyapunov Exponent. Their results clearly indicate the presence of nonlinear dependence in high frequency FTSE returns. However, they found very little evidence to support the view that returns could be characterized by a low dimension chaotic process.

Barkoulas, Baum and Travlos (2000) tested the Athens Stock Exchange (ATS), an emerging capital market, for the presence of fractional dynamics, or long memory, in the return series through application of the spectral regression method on weekly data over a ten year period. The obtained results suggest that there is significant evidence of fractional dynamics with long-memory features in the stock return series of ATS and price movements appear to be influenced by realizations from both the recent past and the remote past and which is contradictory to random walk model that states future returns are unpredictable.

Abhyankar, Copeland and Wong (1997) measure six indices for the three months of September through November 1991. They use a one-minute sampling frequency for the FTSE-100, the DAX, and the NIKKEI; a 15 second sampling frequency for the S&P 500, and transactions for the FTSE and S&P futures indices. They test with the BDS statistic and the Lee-White-Granger 1993 test which is a neural-network based test. They find nonlinearity for all six series, which is partly due to volatility clustering. They also try to document that these returns follow a deterministic system and are unable to do so. They conclude that chaotic system does not exist at all in this data, or that is masked by an exceptionally strong stochastic process.

Kohers and Pandey (1997) inquired for the existence of chaotic mechanisms in returns of stock portfolios segmented by firm size and stock exchange by examining the returns in New York Stock Exchange, the American Stock Exchange, and Over-the-Counter Market. Three portfolios, each one consisting of companies with approximately equal capitalization, selected from each one of the three major stock exchanges are examined for the sample period from 1973 through 1990 by using the BDS statistics to determine if the null hypothesis of independent and identical distribution is violated and by using three moment test to distinguish deterministic nonlinearity from stochastic nonlinearity. The following conclusions had been reached: (1) The NYSE large-firm portfolio is positively driven by a chaotic deterministic process, (2) The AMEX large-firm portfolio was not stationary over the sample period, and hence, could not be tested for the presence of chaotic dynamics; (3) All other examined portfolios do not display any signs of being influenced by a nonlinear-in-mean system.

Bozdağ (1998), in his doctoral dissertation called "Chaos Analysis: An Application of Financial Sector", has aimed to search if there are any evidence for nonlinearity and chaos in Istanbul Stock Exchange index and individual stocks' return indices. He analyzed 130 individual stocks. His study has addressed three major questions: "Is there any long-term memory in Istanbul Stock Exchange (ISE) Index and in individual stock returns?", "Is there a nonlinear structure in the process generating returns on the individual stock returns?" and if so "is there any evidence that the process may be chaotic?". He applied Rescaled Range Analysis and found Hurst exponents to prove if there is a long-term memory, applied Correlation Dimension test to prove if there is any nonlinearity in ISE index and finally applied Lyapunov Test to prove if the nonlinearity in indices has shown chaotic behaviors. His conclusions are that random walk theory is not valid for almost 50% of individual stocks and have long-term memory; ISE index and individual stock indices does not show a stochastic behavior and all time series are deterministic. The Lyapunov exponents of all individual stocks are positive. He also proved that ISE composite index has shown chaotic behavior and can be modeled with low dimensional nonlinear models.

Barkoulas and Travlos (1998) investigated the existence of a deterministic nonlinear structure in the stock returns of the Athens Stock Exchange (Greece), an emerging capital market. The analysis utilizes the concepts of correlation dimension and Kolmogorov entropy by using Greek daily stock returns based on the closing prices of a value-weighted index comprising the 30 most marketable stocks during the period 1988-1990 in Athens Stock Exchange (ASE30). They found that the behavior of Greek stock returns may be consisted with a nonlinear stochastic process and stronger evidence in support of chaos was not obtained.

Harris and Küçüközmen (2000) investigated the dynamic behavior of Istanbul Stock Exchange (ISE) and employed the BDS test to detect nonlinear dependence in daily observations on the ISE composite stock price index (the period from 4th January 1988 to 25th October 1999- a total of 3081 observations). They found that daily ISE returns display significant nonlinear dependence.

McKenzie (2001) develops and compares the close returns test to the BDS test in evaluating the nonlinear behavior of several major national stock market indices. He finds that while the close returns test indicates that the index return data are not chaotic, and close returns test reveals more nonlinearity than the BDS test does.

Ammermann and Patterson (2001) use the Hinich bispectrum and the Hinich Patterson statistics to test the daily closing values for stock market indices from six different stock markets across the world: the Dow Jones Industrial Average(DJIA) from the New York Stock Exchange, the Taiwan Stock Exchange Weighted Stock Index (Taiex) from the Taiwan Stock Exchange, the Nikkei 225 Stock Composite Index from the Tokyo Stock Exchange, the Hang Seng Index from the Hong Kong Stock Exchange, the Singapore Straits Times Industrials Index from the Singapore Stock Exchange, and the London Stock Index from the London Stock Exchange. Observations were taken from January 1982 through February 1993 for each index, but due to differences in holidays across countries, as well as the existence of Saturday trading on the Taiwan and Tokyo Stock Exchanges, the total number of observations range from 2750 for the Hang Seng Index to 3142 for the Taiex. Rates of return are calculated by taking the logarithmic differences in closing values between trading days. They find episodes of nonlinearity for essentially all of the indices and the stocks. They also studied the existence of nonlinearity on the Taiwan Stock Exchange include the daily closing prices for the total of 258 common and preferred stocks that traded on the exchange at some time between January 1984 and December 1992. Most notable about their contribution is that Taiwan's stock market has certain fundamental characteristics that are different from the NYSE. The Taiex prohibits short selling, has no other derivative markets, matches trades electronically rather than through open outcry, allows a very small amount of foreign investment (through mutual funds only), and during this period enforced daily price limit changes of 3-7%. The Taiex is also among the most heavily traded markets in the world. The implication of these results is that nonlinear serial dependencies do play a significant role in the vast majority of stocks trading on the Taiwan stock exchange.

Aydoğan, K. and Booth, G. (2001) has analyzed two hundred randomly selected stocks for the 18.5 year period beginning from December 1980 and ending July 1992. For each stock 965 weekly rates of return are computed from the CRSP daily stock returns. Weekly rates of return for the CRSP value-weighted index are also obtained. In the case of common stock returns, the weight of evidence suggests that either long-term dependence (i.e. significant autocorrelation between observations means that observations are not independent as suggested by random walk theory) is not prevalent or that is too small to be accurately measured by rescaled range analysis. The conclusion holds for individual stocks as well as the market.

Çinko (2001), in his paper called "Non-linearity Test for Istanbul stock exchange" has tried to prove whether economic relations are nonlinear and nonlinear models can be used to forecast time series such as returns. He applied three tests which are BDS test, Hinich Bispectral test and NEGM Lyapunov exponent test to Istanbul Stock Exchange Index. The data was daily closing index between 02/01/1989 and 26/01/2001, which is 2995 observations, obtained from Central Bank of Turkey. He has rejected the presence of linearity for ISE return by the BDS. This can be considered to be significant evidence against linearity since this test is accepted to be powerful tests against any departure from linearity. By NEGM test he could not find evidence on chaos. Furthermore, by Hinich's bispectrum test he could not find any conclusion that supports a third order nonlinear dependence. It can be concluded that he has found sufficient evidence for nonlinearity in the ISE daily returns but the form of nonlinearity can be any type except chaotic or third order nonlinear.

Chu (2003) investigated the daily return data from the Shanghai Stock Exchange Index and the Shenzhen Stock Exchange Index whether there are non-random, nonlinear and chaotic characteristics by employing various tests from chaos theory. The Hurst exponent in R/S analysis rejects the hypothesis that the index return series are random, independent and identically distributed. The BDS test provides evidence for non-linearity and the estimated correlation dimensions provide evidence for the deterministic chaotic behaviors.

2.3. EMPIRICAL RESEARCH ON OTHER ASSETS

Brockett et al. (1988) analyzed foreign exchange rates and found that the linear models previously postulated with the random walk hypothesis do not fit the data. They applied Hinich bispectral tests to find evidence of nonlinearity in the 30-day forward dollar/yen exchange rates (sampled from 1981-1983) and the corresponding spot rates. They concluded that the spot, forward, log-spot and log-forward time series are nonlinear.

Hseih (1989) examines the daily Hsieh (1989) examines the daily closing bid prices of five foreign currencies (in terms of U.S. dollars): the British pound, the Canadian dollar, the German mark, the Japanese yen and the Swiss franc, from 1974-1983 by using BDS test and third-order moment test. His article shows that daily exchange rate changes are not independent of past changes. Although there is little linear dependence in the data, the BDS test and autocorrelations of the squared data detect strong nonlinear dependence. Also, evidence from third-order moments indicates that the nonlinearity is likely to enter through variances rather than through means.

Serletis and Gogas (1997) tested for deterministic chaos in seven East European black market exchange rates, using monthly data from January 1955 through May 1990. They used three non-parametric inference methods, the BDS test for whiteness, the Lyapunov exponent estimator of Nychka et al and the Lyapunov exponent estimator of Gencay and Dechert. They concluded that there is evidence consistent with a chaotic non-linear generation process in only two out of seven series.

In his thesis, Çinko (2000) tested the existence of nonlinearities in daily gold return in Turkey and found that gold return has a nonlinear dependence; and so proved that market efficiency theory says that prices are following random walk can be refuted. In this study, Çinko has used the data between 01.08.1995 and 31.05.2000, with 1193 data point and used five nonlinearity tests common in literature called BDS test, Kaplan test, White Neural Network test, Hinich Bispectral test and NEGM Lyapunov test. He proved that BDS, Kaplan and white tests detect nonlinearity in the data whereas Hinich Bispectral and NEGM Lyapunov exponent tests reject the third order nonlinearity and chaos. His conclusion was gold returns have nonlinearity, however neither chaos nor third order nonlinearity can be found.

3. INCONSISTENCIES AND FAILURES OF USING LINEAR METHODS TO MODEL AND FORECAST OF FINANCIAL MARKETS

"Where chaos begins, classical science stops". (Gleick, 1987, p.3)

Until very recently, scientists have been accustomed to describing the world in terms of "smooth" mathematics with lines, curves, surfaces and volumes and used Euclidean, Galilean and Newtonian vision of the structure and dynamics of universe has reigned for more than two centuries due to being extraordinarily useful and simple techniques. Scientist has generally preferred to use linear models when trying to understand how the universe has processed since linear models show more basic behaviors and so have easier solutions. As an example economists have given priority to linear models which has one solution or equilibrium point.

New science has been questioned that whether the universe is a continuous and predictable simple mechanism and whether the effects of issues such as social and psychological behaviors, esthetics, emotions, spirituality, free will, random happenstance are null. Such variables which can not be rejected in real life has created more complex behaviors and also the data obtained from real life has shown that the system of the world is not a deterministic one with one true solution as it is proposed with linear models. New science has concluded that linear models are an approximation to a more complicated reality, and are mainly made for mathematical and statistical convenience under the assumption that linearity is a reasonable approximation.

In the table below, comparison of main characteristics of classical science with new science can be seen:

Table 3.1

Comparison of Classical Science with New Science

Classical Science	New Science
Orderliness	Disorderliness
Predictable Results	Unpredictable Results
Deterministic	Probabilistic
Scientific orders are valid	Probabilistic orders are valid
Mechanical Universe	A universe in order within disorder
Past data does not influence present or future	Long-memory of past
Simple linear systems	Complex and non-linear systems
Simple mathematical calculation	Complex and hard calculations due to non-proportionality
Reductionist Approach	Integrated and universal approach

Source: Bernice, 1997, p.80

3.1. THE TRADITIONAL APPROACH: DEVELOPMET OF EFFICIENT MARKET HYPOTHESIS

The bedrock of quantitative capital market theory and the past 30-plus years of research in finance have depended on Efficient Market Hypothesis (“EMH”). Capital Asset Pricing Model (CAPM) by Sharpe, Litner and Mossin, the Asset Pricing Theory by Ross and Modern Portfolio Theory by Markowitz and the option pricing model by Black and Scholes and Merton etc. are all based on EMH. One primary reason for the usage EMH so widely that EMH justifies the use of probability calculus in analyzing capital markets. Because if the markets are nonlinear dynamic systems, then the use of standard statistical analysis can give misleading results, particularly if a random walk model is used.

Standard asset pricing models has implied that markets are efficient means that securities are priced so that all public information, both fundamental and price history, is already discounted. In this market, today’s price changes are only because of today’s unexpected news. In an efficient market no extended arbitrage opportunities may exist since all that new information is quickly absorbed in price changes, and also the large number of investors will ensure that prices are fair. According to EMH, price increments in value are unpredictable since all information available is fully reflected in prices and hence information set is useless in predicting the expected price of the asset. This implies that no amount of data mining can predict future prices means that no trading mechanism can consistently beat the market. Hence, for a given level of risk, speculators cannot earn supernormal returns.

Fama (1970, p.385) defined three types of market efficiency, each of which is based on what type of information is reflected in the prices. The weak form simply states that all past information is reflected in current prices. The semi-strong form states that all publicly available information is incorporated in prices, while strong form, states that all information including insider information is included in prices.

In this perfect market, investors are considered rational. They know what information is important, so there can be an equilibrium price because of the collective consciousness of the market. Information is coming randomly, which perturbs the prices to fluctuate unpredictable. Yesterday's information has been reflected in today's prices and then is no longer important. The market does not need memory since the present prices have contained all the information. So market efficiency hypothesis supports that market returns are independent, identically distributed (i.i.d) like a random walk. If returns are independent, then they are random variables and follow a random walk. If enough independent price changes are collected, in limit-as the number of observations approaches infinity-, the probability distribution becomes the normal distribution. Shortly, according to random walk process advocated with EMH, return distribution is identical and independent, expected average return is zero and variance of that distribution is constant means that does not change with time. This process has described as a stochastic process called "Brownian Motion" with the assumption of normal distribution. This assumption regarding the normality of returns enables the usage of large amount of statistical tests and modeling techniques, which create optimal solutions for decision making. As it can be seen, one primary function of the EMH is to justify the use of probability calculus in analyzing financial markets.

Meanwhile Modern Portfolio Theory ("MPT") was also being developed. Increasing importance of the risk concept in financial markets has caused to look for a risk measurement which is appropriate and well described for pricing financial assets. In this classical portfolio theory, risk is described as "variance of probability returns". Markowitz mean-variance analysis is concerned with how the investor should allocate his wealth among the various assets available in the market. Using variance as a risk measurement has put on the agenda of the subject of determination of the distribution of financial returns since in order to measure the risk with variance, a normal distribution is required.

Sharpe, Litner and Mossin were extended these concepts mentioned above and an asset pricing model called Capital Asset Pricing Model ("CAPM") was introduced. The CAPM combined the EMH and Markowitz's mathematical model of portfolio theory into a model of investor behavior based on rational expectations in a general equilibrium framework. CAPM is only valid within a special set of assumptions. The main assumptions underlying the CAPM are stated below (Peters, 1991, p.21):

1. Investors have homogenous return expectations (beliefs) about asset returns. It means that all investors perceive identical opportunity sets. This is, everyone have the same information at the same time.
2. There are no market imperfections such as transaction costs, commissions, taxes, regulations, or restrictions on short selling.
3. There exists a risk free asset and all investors may borrow or lend unlimited amounts of this asset at a constant rate: the risk free rate, which is usually interpreted as the 90-day T-bill rate.
4. All investors desire Markowitz mean/variance efficiency- that they want the portfolio with the highest level of expected return for a given level of risk, and are risk averse. Investors are therefore rational.

As it can be seen, standard deviation is used as the measure of risk in CAPM and CAPM needs efficient markets and normally or log-normally distributed returns, because variances are assumed to be finite.

The CAPM explained how investors would behave, if they are rational and Markowitz portfolio theory explained why diversification reduced risk. Practitioners have been convinced that CAPM's underlying assumptions did not affect the validity of this model. The EMH became extensively used as a rationale for the Gaussian assumption of log-normally distributed returns.

3.2. FAILURES AND INCONSISTENCIES OF EFFICIENT MARKET HYPOTHESIS

Empirical studies have attempted to prove capital market theory due to its simplifying assumptions and its limitations even by the early founders of that theory. Exceptions or anomalies to the normality assumption were being found in financial markets which proved the deviation from simplifying assumptions of EMH such as volatility, the small firm effect, low Price/Earning effect, January effect, overreaction to extreme bad or good news about a firm and long-term memory. The advent of powerful computers gave ability to researchers in the field of finance to model complex systems which shows nonlinearity or chaotic behavior without relying on simplified assumptions.

Some empirical studies to test of normality are summarized below. Osborne (1964, cited in Peters, 1991) plotted the density function of stock market returns, and labeled the returns "approximately normal": there were extra observations in the tails of the distribution, a condition that statisticians call "kurtosis[*]". Osborne noted that the tails were fatter than they should be in a normal distribution.

Mandelbrot (1963), has stated that large-scale changes in the prices of financial assets has followed by large-scale movements and small-scale changes has followed by small-scale movements proved that the volatility clustering. This situation has proved the most important characteristics of financial variables which are financial variables are not static- they are dynamic and they are not independent.

[*] In probability theory and statistics, kurtosis is a measure of the "peakedness" of the probability distribution of a real-valued random variable. A distibution with positive kurtosis is called "leptokurtic". In terms of shape, a leptokurtic distribution has a more acute "peak" around the mean (that is, a higher probability than a normally distributed variable of values near the mean) and "fat tails" (that is, a higher probability than a normally distributed variable of extreme values).

Fama (1965) found that daily returns were negatively skewed: more observations were in the left hand tail than in the right hand tail. Leptokurtosis conditions have found in the distribution of financial time series with high volatility means that excess peakedness from average and fatter tails.

Sharpe (1970, cited in Peters, 1991) also noted that "normal distributions assign little likelihood to the occurrence of really extreme values. But such values occur quite often".

Campbell, Lo and Mackinlay (1997, p.25) have argued that many aspects of economic behavior may not be linear. At a theoretical level, it has been shown that even very simple economic models often involve a rich variety of dynamic processes, including in some cases the possibility of nonlinear or complex chaotic behavior for some range of parameter values. Experimental evidence and casual introspection also suggest that investors' attitudes towards risk and expected return are nonlinear.

As it can be seen EMH has worked on a limited scale, but it is unable to handle situations that involved multiple interdependent variables and instability (volatility). Because, any system that has interdependence between variables must use nonlinear mathematics and if it is assumed that, markets are nonlinear then the use of standard statistical analysis used in EMH can give misleading results. Chaos theory provides new insights into the anomalies of efficient market hypothesis.

In financial market applications, chaos is often presented in contrast to the EMH (Yunho, 1998, p.18). Chaos theory offers a number of theoretical advantages in analyzing financial markets. Chaotic behavior appears to be random, but it is an erratic (unstable) motion which is generated by a nonlinear deterministic system with relatively few degrees of freedom. Hence, the existing of chaos creates the possibility that profitable trading rule based on nonlinearity may exist at least in the short run, provided the actual generating mechanism is known (Yunho, 1998). Although empirical studies of chaos have encountered severe technical difficulties, there are important reasons for continuing to push the understanding of chaos-like behavior. A similar argument can be made for nonlinear models in general. The control of nonlinear systems may actually be easier than the control of linear ones, because it might take a small push to engender a large change in the system. It should be noted that although interest in chaos is due to its ability to generate output that mimics the output of stochastic systems and thereby offering an alternative explanation for the behavior of asset prices, the possible existence of chaos could be exploitable and even invaluable. If, for example, chaos can be shown to exist in asset prices, the implication would be that profitable, nonlinearity-based trading rules exist (at least in the short run and provided the actual generating mechanism is known). Prediction, however, over long periods is all but impossible, due to the sensitive dependence on initial condition property of chaos.

Development of EMH has based on certain questionable assumptions which are described in the previous section. In this section, the issue of what went wrong with EMH has tired to be explained.

a. Concept of equilibrium-fair price: Econometric analysis assumes that if there are no outside or exogenous influences then a system is in balance, shortly supply equals demand. If an exogenous factor affects the system, system react that factor to revert the system in equilibrium again in a linear fashion. Also, it has assumed that there is free-market economy which is an economy there is no control or intervention. In this market, which is called efficient market, assets are fairly priced according to information available and neither buyers nor sellers can have profit. But, markets are functioning with other considerations rather than fair price and volatiles with the emotional forces and human tendencies far from equilibrium and even it can be said that there can not be a healthy market without volatility. Buyers and sellers will only trade at a mutually advantageous price: equilibrium. Chaos theory has received an endogenous explanation to the economic fluctuations, rather than exogenous approach to economic fluctuation, based on the assumption that economic equilibrium are determinate and intrinsically stable, so that in the absence of continuing exogenous shocks the economy tend towards a steady state, but because of stochastic shocks a stationary pattern of fluctuations are observed (Barnett, Medio&Serletis, 1997, p.36-37).

b. Treatment of Time: efficient market hypothesis ignores the time and advocates that markets have no memory or only limited memory of past means that history is irrelevant. As a first consequence the future would be unrelated to the past or the present, with no possibility of identifying trends or cycles. Forecasts were maybe relevant in only a short time frame because a small change in one variable seemed to have a much bigger impact than the random walk theory would suggest. The missing point of that theory is human decision making. Traders are influenced by what has happened and their expectations of the future have shaped by their recent experiences. Real feedback systems involve long-term correlations and trends, because memories of long-past events can still affect the decision made in the present. We can say that time dependent feedback mechanisms are symptomatic of nonlinear dynamic systems.

c. Gaussian normal distribution of stock return: Because price changes are independent then they are random variables and follow a random walk and the distribution of changes are normal with a stable mean and finite variance. But, variables in the market is not independent, indeed economic variables are interdependent-they have influence on one another- and internal to the system.

d. Linear Modeling: The efficient market hypothesis is using the old linear paradigm and advocates that the investors react only to today's information, which is basically a linear fashion. In linear models cause and effect are clearly defined. This means that cause is proportional to effect and to examine the relationship between two variables, other all factors should be hold constant. However, the use of linear tools to model a non-linear world is questionable. In non-linear models there is no proportional cause and effect relationship between variables and no simplifying assumptions can be made since variables are interdependence and there is complex relationship in system.

e. Rational Investor: Rational investors have access to all available information, using it to bid prices up and down until equilibrium is reached. The logical decision is always to pick the asset with the highest expected return. There are no rational investors in financial markets. People often follow the crowd and trade with the trend rather than behaving in a rational manner (bubbles-where prices move more than is warranted by the underlying factors- in financial markets, herd effect-markets can be subject to bouts of excessive optimism followed by waves of excessive pessimism-). For example, even the stock price is rising; investor might still want to wait for more information coming to confirm the rising. This kind of nonlinearity behaviors is widely observed in financial markets since in real financial markets investors can not master all the information and so delays or mistakes always exists.

f. Market Frictions: The frictions include transaction costs, regulations, commission fees, bid-ask spread and other different costs. In EMH, it is assumed that there is no transaction or any other kind of costs or treated them in the way that they could never cause big differences. However, if these costs are in a non-linear fashion they could make big differences, so these markets may have more chaotic patterns than the others. Option markets, derivative markets and some exotic financial productions are created to with the hopes to lower costs. The goal is achieved, but one of the byproducts is high volatility.

g. The term structure of Volatility: Another basic assumption needed to apply the normal distribution involves the term structure of volatility. Variance is used to measure volatility and it scales according to the square root of time. For example, in the normal distribution the variance of 5-day returns should be five times the variance of daily returns or if standard deviation is used as the measure of volatility daily standard deviation is multiplied by the square root of 5. This scaling feature of the normal distribution is referred to as the $T^{1/2}$ Rule, where T is the increment of time. However, many empirical papers report that the volatility increases slower (faster) than the square root of time in the short (long) term, reflecting significant deviance from random walk hypothesis (Peters, 1994, p.257). Turner and Weigel (1990, cited in Peters, 1991, p.31) found that monthly and quarterly volatility were higher than they should be, compared to annual volatility, but daily volatility has lower than it should be. Shiller (1989, cited in Peters, 1991, p.31) notes also that rational investors' valuation of stocks would be based on expected dividends from owning the stocks; however, prices are much too volatile to be due to changes in expected dividends. This excessive market volatility challenged the idea of rational investors and the concept that by having large numbers of investors, the achievement of market efficiency would be ensured.

h. Random Walk: Since returns must be random variables that are independent and identically distributed, and because markets were known to be large systems with many degrees of freedom (means many investors), prices must be fair for only then would they reflect all publicly available information. Thus, price changes could only come about by unexpected news.

4. NONLINEAR DYNAMICAL SYSTEMS and CHAOTIC BEHAVIOR

This section will briefly recall the basic mathematical definitions and properties of some concepts related to chaos theory and nonlinear dynamic systems, insofar as they are relevant to applied time series analysis.

4.1. BASIC THEORETICAL CONCEPTS

4.1.1 Dynamical Systems

A dynamical system consists of a set of possible states, together with a rule that determines the present state in terms of past states (Alligood, Sauer& Yorke, 1996). A dynamical system changes over time with this rule. A simple example for a dynamical system is presented below (Hilborn, 2000, p. 17):

State→ x: The population level of bacteria in a laboratory culture

Rule that changes with time →$f\,(x_{n+1}) = 2(x_{n-1})$, the population one hour later

If the culture has an initial population of 10,000 bacteria, then after one hour there will $f\,(10{,}000) = 20{,}000$ bacteria, and so on.

It is required that the rule should be deterministic, which means that present state (population for this example) can be determined uniquely from the past states. No randomness is allowed in this definition of a dynamical system. So, it could be said that while analyzing the time series data of a system to determine the behavior of the system, three ingredients of the system should be known. These are: the time-evolution equations; the values of the parameters describing the system and initial conditions (Hilborn, 2000, p.6). A system is said to be deterministic if knowledge of these three components, in principle completely determine the subsequent behavior of the system. Mathematically, deterministic dynamical systems can be defined as "deterministic difference or differential equations on n-dimensional Euclidean space, more specifically deterministic dynamical system on n-dimensional space is a system of n difference or differential equations in n variables" (Brock, 2000, p.1-2).

In fact, precise knowledge of these three ingredients, which describe the system, could be a difficult problem, even impossible. Instead of a deterministic dynamical system, a possible explanation is a *random or stochastic process*. A typical realization of such a process could be achieved basically by flipping a fair coin continuously.

There are two different approaches to explain the behavior within the framework of a dynamical model with respect to random process. The traditional approach is that time evolution of the series can be explained by a stochastic process. In this case, the observed irregular behavior is explained by the influence of external random shocks which are exogenous to the system such as war or catastrophe (Hsieh, 1991, p.2). Because these outside influences are changing in uncontrolled ways, the system's behavior appears random. Nonlinear dynamics and chaos provides an alternative explanation for this apparent randomness: "systems in real life are made of billions of parameters means that these complex systems have many degrees of freedom and so lack of control. It is the activity of these many degrees of freedom that leads to the apparently random behavior" (Hilborn, 2000, p.7). Put another way, this theory argues that fluctuations can be caused endogenously and chaotic behavior also appears to be random. It should be noted that only low complex chaotic behavior which is generated by a nonlinear deterministic system with relatively few degrees of freedom can be detected since highly complex chaotic process may never be distinguished from the randomness (Hsieh, 1991, p.6).

4.1.2 Classification of Dynamical Systems

For investigating dynamical systems it is necessary to specify some characteristics that provide a subdivision into special classes of dynamical systems.

An important characteristic of a dynamical system is whether it is discrete or continuous.

In *discrete dynamical systems*, the state of the system has been determined in different time intervals. The state of the system is changed only after a finite interval according to the rule defined means also that time can take on only integer values.

Mathematical model for biological population growth model can be given as an example to discrete dynamical systems. The model can be built up by taking the initial number of species as N_0 and a difference equation to guess the number of species in the time N_1 as $N_1 = AN_0$, where A is some number that depends on the conditions of the environment (Bozdağ, 1998, p.6-7). If N_0 is taken the initial value of the system and A is a constant, the number of the species in time will be;

$N_1 = AN_0$

$N_2 = AN_1$

....

$N_{t+1} = AN_t$

......

Time path of the values of $N_0, N_1, N_2, \ldots N_t \ldots$ will be the motion of this dynamic system. The graphical presentation of this population growth model for A=1.5 is below:

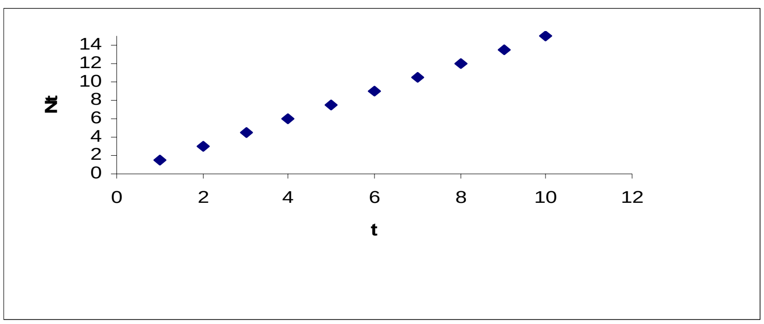


Figure 4.1: Graphical Presentation for Example of Discrete Time Series: Biological Growth Model

In *continuous dynamical systems,* state of the system is determined continuously in every point of time (Bozdağ, 1998). In the case of a continuous time, the time step is infinitesimally small. Thus, differential equations are used to define the transformation of the system as mathematically. Biological growth model can be converted into a continuous dynamic system. The differential equation is;

$$dN / dt = AN$$

The graphical presentation of this population growth model for A=1.5 and N_0=1 is shown below:

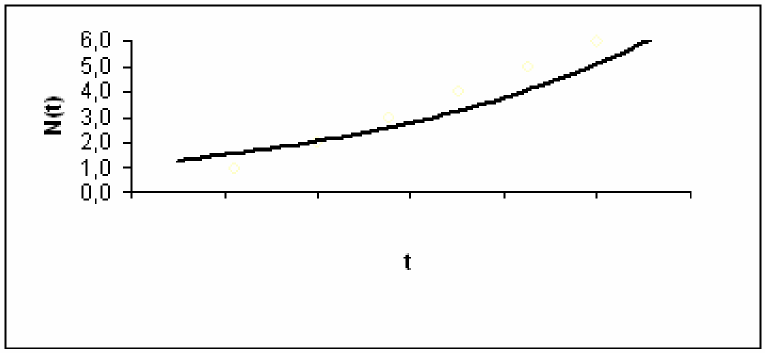


Figure 4.2: Graphical Presentation for Example of Continues Time Series: Biological Growth Model

The most important classification of dynamical systems for chaos analysis is that "*linear*" and "*nonlinear*" dynamical systems.

Linear dynamics mathematically models the change in system behavior with linear difference or differential equations which is a "straight line" change and includes simple cause and effect relationship where a change in A causes a proportional change in B. For example, if we give the system a "kick" and observe a certain response to that kick, then if we kick the system twice as hard, the response will be twice as large. Deviation from the linear models have considered as "error" or "noise" in classical science (Gürsakal, 1999).

Non-linear dynamics is concerned with the study of systems whose time evolution equations are nonlinear; that is, the dynamical variables describing the properties of the system appear in the equations in a nonlinear form.

Illustration of linear and nonlinear systems is presented below;

In classical mechanics the behavior of a system consisting of a point particle with mass *m* and subject to a force F_x acting in the *x* direction and constrained to move in only the x direction is given by the Newton's Second Law of Motion:

$$F_x(x,t) = ma = m(d^2x/dt^2) = -kx \quad \text{or} \quad \frac{d^2x}{dt^2} = -\frac{k}{m}$$

This equation is linear in x and in the second derivative of x. "k/m" rate gives the displacement from the equilibrium position of the system with the affect of some degree of oscillation.

When the force F_x is given by a more complicated x dependence, such as $F = bx^2$, then the time evolution equation has become as;

$$\frac{d^2x}{dt^2} = -\frac{b}{m}x^2$$

and the system behavior has became nonlinear since the x position of the particle appears in the equation squared.

Chaos is only one type of behavior exhibited by nonlinear systems. All chaotic systems are nonlinear, but not all nonlinear systems are chaotic. The field of study is properly and more generally called nonlinear dynamics, the study of dynamical behavior, which is the behavior in time, of a non-linear system.

4.1.3 Nonlinearity and Chaos

The theory of non-linear dynamics allows us to describe and classify complex behavior of systems and theory of chaos allows us to see an order and universality that underlies these complexities. Chaos theory is considered as a new paradigm for looking at events which happen in the world differently from the more traditional, strictly deterministic view which has dominated science from Newtonian times. One of the central concepts of chaos theory is that complex nonlinear systems are inherently unpredictable-but at the same time; chaos theory also ensures that, the way to express such an unpredictable system lies not in exact equations, but in representations of the behavior of a system and it is generally quite possible to model the overall behavior of a system. Thus, chaos theory lays emphasis not on the disorder of the system--the inherent unpredictability of a system--but on the order inherent in the system--the universal behavior of similar systems.

For nonlinear systems, a small change in a parameter can lead to sudden and dramatic changes in both the qualitative and quantitative behavior of the system. For one value, the behavior might be periodic, for another value only slightly different from the first; the behavior might be completely a-periodic. Chaotic behavior may arise from those sudden and dramatic changes in nonlinear systems. The noun *chaos* and the adjective *chaotic* are used to describe the time behavior of a system when that behavior is a-periodic and instable. Instable a-periodic behavior is highly complex. It never exactly repeats and it continues to manifest the effect of any small perturbation. Due to these characterizations of system behavior, change in system behavior looks apparently random or “noisy” (Hilborn, 2000, p.6).

According to the complexity argument, Kellert (1993, cited in Bozdağ, 1998) has described the chaos as “The qualitative study of instable and a-periodic behavior in deterministic nonlinear system.” The problem is that the future of a chaotic system is indeterminable in the long-term even though the system is deterministic and described by differential equations, which do not contain any random function. This is due to a small change in initial conditions leads to grossly different long-term behavior of the system. Brock (2000) has also asserted that “*Chaos theory* is the study of deterministic difference (differential) equations that display “Sensitive Dependence on Initial Conditions” to generate time paths that look like random behavior (Brock, 2000)”.

We can summarize the reasons of unpredictability in most of the nonlinear dynamic systems as follows:

- There is no analytical solution for the system behavior

- Initial conditions can not be determined with certainty (Uncertainty of Measurements)

- A small change in initial conditions leads to grossly different long-term behavior of the system (Sensitive Dependence on Initial Conditions)

In the literature on chaos, the search for chaos is in reality a search for "low" dimensional chaos, since knowing that data has been produced deterministically from high dimensional chaos is not useful.

4.1.4 Detecting Chaos: Basic Mathematical Concepts in Chaos Analysis

Fractals

An interesting feature of systems that show chaotic behavior is "self-similarity" which is explained by the fractal geometry. The term fractal was first coined by Benoit Mandelbrot and he defined fractals based on topological dimensions. His definition is "a fractal is an object in which the parts are in some way related to each other" (Yunho, 1998, p.25). Another definition is that a *fractal* is a geometric figure which is self-similar to itself at different scales (Peters, 1991, p.53). Most natural systems and time series are described by fractals in terms of simple rules. One of the most easily perceived natural fractals is a tree (Yunho, 1998, p.25). Trees branch according to a fractal scale. Each branch has its smaller branches, which are similar to the whole tree in a qualitative sense. Another example is fractal time series of stock prices. Self-similarity in a time series plot of daily, weekly, and monthly stock returns can be easily found. Furthermore, the closer look at the time series meaning that the smaller the time increment, the more detail can be seen which creates complexity. Fractal geometry models objects which are infinitely complex. Therefore, it can be said that fractals are the geometry of chaos.

There are two types of fractals: deterministic and random (Peters, 1991, p.53). Deterministic fractals are generally symmetric. Random fractals do not necessarily have pieces that look like pieces of the whole. Instead, they may be qualitatively related. In the case of time series, fractal time series are qualitatively self-similar in that, at different scales, the series have similar statistical characteristics. Although, this property has similar to the normal distribution, fractal time series can have fractal dimensions, but the normal distribution has an integer dimension of 2, which changes many of the characteristics of the time series.

Fractal dimension allows us to measure the complexity of an object. Most real objects are so jagged and irregular that they do not exactly fit simple classification in one, two and three dimensions. Generally, objects (or time series) are embedded in a space that is larger than its fractal dimension. For example, a crumpled ball of paper is embedded (the space where the object is placed is called the *embedding dimension)* in a three-dimensional space, although it does not fill that space. The *fractal dimension*, which describes how an object (or time series) fills its space, is the product of all the factors influencing the system that produces the object (or time series).A fractal time series will fill its space unevenly because its parts are related, or correlated meaning that each point is correlated with the point plotted before it. Correlation holds points together in a fractal time series, but there are no correlations to hold the points together in a random series. A fractal time series is separated from a pure random series with the help of Hurst Exponent ("H"), which will be explained in detail later. Mandelbrot (1972) has shown that the inverse of H is the fractal dimension. A random walk, with H=0.5 since each event is equally likely to occur, would have a fractal dimension of 2 and it is truly two-dimensional and would fill up a plane. On the other hand, in a persistent time series which is described as fractional Brownian motion when $0.5<H\leq 1.0$, for example when H=0.7, fractal dimension is 1/0.7 or 1.43 which is not an integer value.

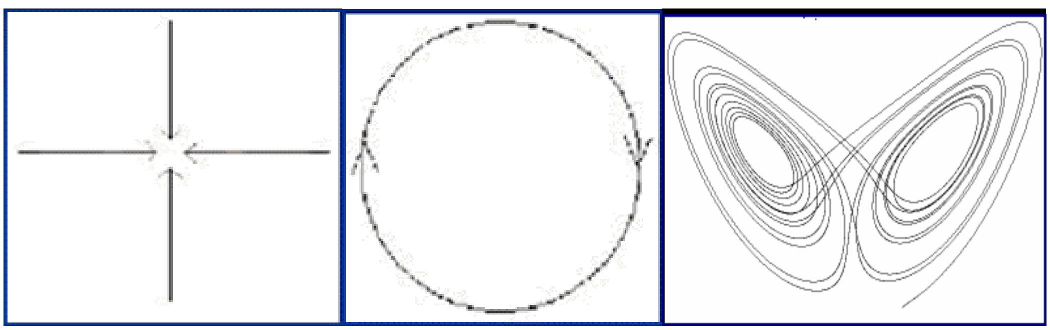
Phase Space (or State Space)

Phase space is an important mathematical tool to present the dynamic systems graphically. It is a graph that shows all possible states of a system. In phase space, every degree of freedom or parameter of the system is represented as an axis of a multidimensional space. For every possible state of the system, or allowed combination of values of the system's parameters, a point is plotted in the multidimensional space. If a system has three descriptive variables, the phase space is plotted in three dimensions, with each variable taking one dimension. A phase space also may contain very many dimensions. For instance, a gas containing many molecules may require a separate dimension for each particle's *x*, *y* and *z* positions and velocities as well as any number of other properties.

Attractors

The evolution of a nonlinear, dynamical, complex system can be marked by a series of phases, each of which constrains the behavior of the system to be in consonance with a reigning attractor(s). In chaos theory, *attractors* are states towards which a system may evolve when starting from certain initial conditions. An attractor, A, is a subset of the n-dimensional Euclidean phase space. The dynamics of the system as well as current conditions determine the system's attractors. When attractors change, the behavior in the system changes because it is operating under a different set of governing principles. Trajectories originating at points on the attractor remain there forever; trajectories based at points not on the attractor, but within a region called the attractor's *"basin of attraction"* approach the attractor to an arbitrary degree of closeness (Hilborn, 2000, p.22). In the case of price data, at an attractor might be equilibrium price level or particular price patterns.

The three primary attractors are the point attractor, limit cycle, and chaotic or strange attractor. A *fixed-point attractor* exists if a system's equilibrium tends to a stable, single-valued point. Its phase space will always be drawn to the point where velocity and position are equal to zero. As a point it represents a very limited range of possible behaviors in the system. For example, in a pendulum, the fixed point attractor represents the pendulum when the bob is at rest. This state of rest attracts the system because of gravity and friction. In case a *limit cycle (periodic) attractor* exists, the system over time, tend to a repeating sequence of states- a periodic orbit. The periodic attractor represents more possibilities for system behavior than the fixed point attractor. A simple pendulum with periodic replenishments of energy is an instance of such a system. In an organization, a periodic attractor might be when the general activity level oscillates from one extreme to another. While the above two types of attractors are easy to detect, a third type of attractor, the *chaotic attractors* (also called fractal attractors or strange attractors), have generated a lot of interest in recent times. Any chaotic time series has chaotic attractors. Thus detecting chaotic attractors would indicate the presence of chaos. With a chaotic attractor, equilibrium applies to a region, rather than a particular point or orbit; equilibrium becomes dynamic (Peters, 1991). For a chaotic attractor in phase space, *the points never repeat themselves and the orbits never intersect*, but both the points and the orbits stay within the same region of phase space meaning that if a system starts with its initial condition in the attractor's basin of attraction, it eventually ends up in the set (means that the attractor is bounded to the phase space). Also once a system is on an attractor, nearby states diverge from each other exponentially fast. Thus any noise or error in measurement gets amplified rapidly and beyond a point the system becomes unpredictable which shows that a chaotic attractor portrays the characteristic of sensitive dependence on initial conditions found in chaos. And most often, chaotic attractors display elegant symmetric structures with self-similarity at different scales (fractals), meaning that the structure of a chaotic attractor is fractal. The Lorenz attractor is an example of strange attractors.



Point Attractor Limit Cycle Attractor Lorenz Attractor

Source: http://sprott.physics.wisc.edu/lectures/monkey/sld008.htm

Figure 4.3: Types of Attractors

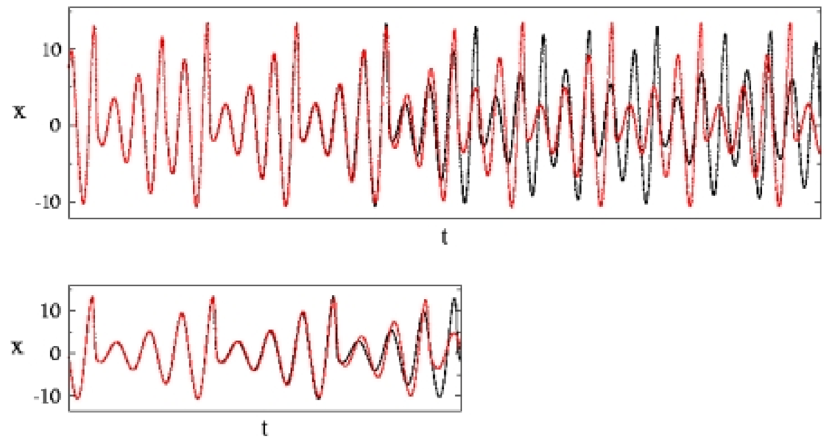
The best known method for measuring attractors is the Lyapunov function which will be explained in detail later. The Lyapunov exponent in a system provides a way of measuring the conflicting effects of stretching, contracting, and folding in the phase space of the attractors. They give a picture of all the properties of a system that leads to stability or instability (Gleick, 1997, p.253). The Lyapunov exponent "quantifies the average growth of infinitesimally small errors in the initial point". These small errors cause instability when continuously fed back into a system.

4.1.5 Important Features of Chaotic Behavior

With the explanations in previous sections, some important features of a nonlinear deterministic dynamic system (chaos) can be summarized in the captions presented below:

1- Sensitive dependence on initial conditions: The effect of the sensitive dependence of initial conditions on the behavior of nonlinear systems has been expressed in an elegant metaphor known as the butterfly effect (Hilborn, 2000). Lorenz has pointed that if the atmosphere displays chaotic behavior with sensitive dependence on initial conditions, then even a small effect, such as the flapping of a butterfly's wings would render our long-term predictions of the atmosphere completely useless.

It means that seemingly insignificant adjustments to a system will be compounded over a time and can dramatically change the overall behavior of a system. In other words, initial points will diverge exponentially. However, as stated before if the initial conditions were to be chosen precisely, a system is completely predictable, but this is where the problem has lied. Our ability to choose exact numerical values for conditions is limited since computers deal with a finite set of numbers. For example, approximating a past stock price to two decimal places instead of four for ease of computation, this action may drastically affect the value of the predicted future price of the stock. In Figure 4.4, divergence of a time series when the initial conditions change in an extremely little:



Source: http://complex.upf.es/~josep/Chaos.html

Figure 4.4: Sensitivity to Initial Conditions: In red, x $_{(t)}$ under initial conditions (0.1, 0.2, 0.3). In black, x $_{(t)}$ under (0.10000005, 0.2, 0.3)

2- Long term behavior is difficult or impossible to predict: An important result of the butterfly effect is that long term prediction is impossible in a system which displays chaotic behavior. To predict the long-term behavior of a real system besides equations that describe the system, initial condition should be known. Even very accurate measurements of the current state of a chaotic system become useless indicators of where the system will be. One has to measure the system again to find out where it is.

3- *There are critical levels* under certain conditions, and at certain times, meaning that more than single equilibrium exists: Critical levels are the values of control parameters where the nature of a nonlinear dynamic system changes. The system can bifurcate or it can make the transition from stable to turbulence behavior.

4- Long-term correlation and trends from feedback effects: As being feedback systems, the current outputs act is inputs to the next state; as a result these nonlinear dynamic systems have a memory element. In regard to financial markets, it is this memory element which allows markets to take into account past events. This memory element suggests the existence of market trends or cycles.

5- *The system has fractal structure:* a time series of returns that, smaller increments of time, will still look the same and will have similar statistical characteristics.

4.1.6 Examples of Nonlinear Deterministic Dynamic Models

Tent map and Logistic map are presented as typical examples of these models.

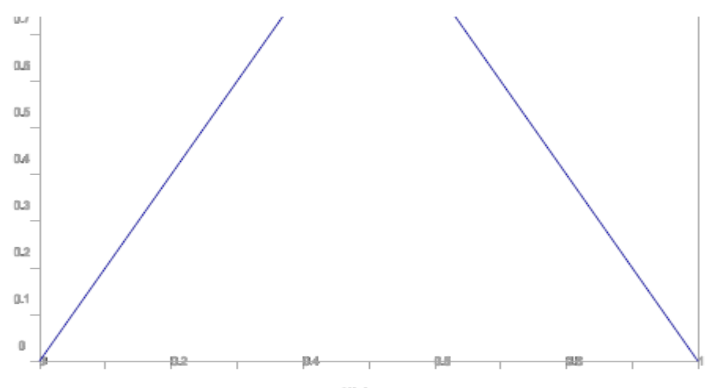
4.1.6.1 Tent Map

The tent map is the simplest dynamical system harboring deterministic chaos (Hsieh, 1991, p.3). The tent map's definition is as follows: The state is a single variable, X_t, which takes on values ranging from 0 to 1. The dynamical rule to generate the sequence of numbers X_t is (Hsieh, 1991, p.3),

$$X_t = 2\,X_{t-1} \quad , \qquad \text{if } X_{t-1} \leq 0.5$$

and $$X_t = 2\,(1-X_{t-1}), \qquad \text{if } X_{t-1} > 0.5$$

The graph of X_t versus X_{t-1} is shaped like a "tent". X_t is a nonlinear function of X_{t-1}.



Source: http://acm.cs.umn.edu/~look/chaos/tentmap.gif

Figure 4.5: Tent Map

The figure clearly shows that a unique future state correspond to each past; the past is mapped into the future. There are four important properties of the tent map (Yunho, 1998, p.23). First, X_t fills up the unit interval [0,1] uniformly as it goes to infinity; second, any small error in measuring the initial X_0 will be compounded in forecasts of X_t exponentially fast; third, X_t appears stochastic even though it is a deterministic process; fourth, X_t can have a series of small increases, and then it suddenly declines sharply.

4.1.6.2 Logistic Map

The logistic equation was originally developed to build a model for population dynamics in ecology (Yunho, 1998, p.24). The logistic equation is an extremely simply nonlinear differential equation which consists of a linear part, $k\ X_{t-1,}$ and a nonlinear part, $-k\ (X_{t-1})^2$.

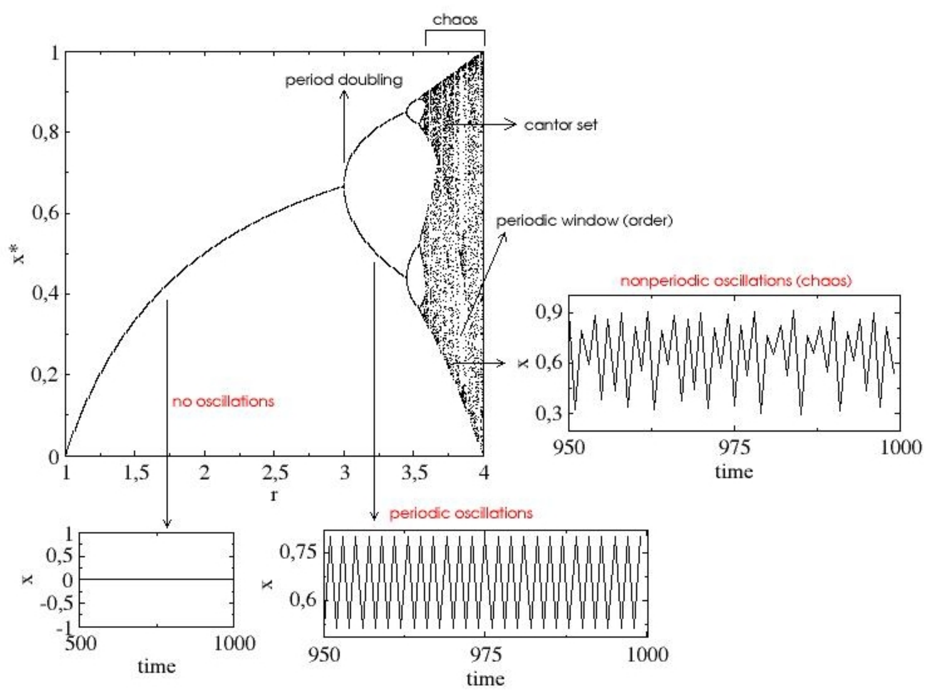
The logistic map is slightly more complex than the tent map. Again, a X_0 is selected between 0 and 1, and generate the sequence of X_t according to:

$$X_t = k\ X_{t-1}\ (1- x_{t-1}),$$

$$=k\ X_{t-1} - \mathrm{k}\ (X_{t-1})^2$$

where k is between 0 and 4. For small values of k, the system is stable and well behaved. But as the value of *k* approaches 4, the system becomes chaotic. The logistic map adds a property to chaotic behavior that the dynamics of a system depends on a parameter such as *k*. For some values of the parameter, the dynamics may be simple, while for other values, the dynamics may be chaotic (Hsieh, 1991, p.4).

In Figure 4.6, the values of x(t) for various values of the growth parameter k, which is the sole control parameter of the logistic process, has shown. At low values of the growth rate, x(t) is firstly approaches a stable value. As the growth rate is increased and passed 3, the line broke in two. Instead of settling down to a single value, it alternates between two fixed values. As the parameter rose further, the line doubled (bifurcated) again and it eventually at a special critical value the logistic system falls into an infinite-period limit cycle, which is chaos.



Source: http://complex.upf.es/~josep/Chaos.html

Figure 4.6 Bifurcation Diagram of Logistic Parabola

Other examples for nonlinear deterministic dynamic models are the pseudo-random number generators, the Henon map and Lorenz map (Yunho, 1998, p.24).

4.2. QUANTITATIVE TECHNIQUES TO DETECT NONLINEARITY AND CHAOTIC PATTERNS IN TIME SERIES

In recent years, there has been growing interest in testing for both chaotic and non-chaotic nonlinearity in economic and financial data. A number of researchers have recently focused on testing for nonlinearity and chaos in particular time series. Although the analysis of macroeconomic time series has not yet led to particularly encouraging results due to small samples and high noise levels for most macroeconomic series, the analysis of financial time series has led to results which are as a whole more interesting and reliable (Barnett et al. 1997, p.56). This is probably due to the much larger number of data available and their superior quality.

In this thesis, three inference methods to test for nonlinearity and deterministic chaos have been used: the Rescaled Range Analysis, the BDS test, the Lyapunov Exponent estimator of Nkhya et al. All three of these tests are purported to be useful with noisy data of moderate sample sizes.

4.2.1 Rescaled Range (R/S) Analysis

Rescaled Range Analysis (R/S) is a tool for studying long-term memory and fractality of a time series. The R/S analysis was first introduced by hydrologist Hurst in 1951, who began working on the Nile River Dam project. He struggled with the problem of reservoir control. The problem was what policy of discharges could be set, such that the reservoir never overflowed or emptied. In constructing the model he is assumed that the uncontrollable part of the system-the influx of water from rainfall-followed a random walk. When Hurst decided to test the assumption, introduced the new statistic called Hurst exponent ("H"). Hurst measured how the reservoir level fluctuated around its average level over time. He assumed that if the series were random, the range would increase with the square root of time ($T^{1/2}$ rule). To standardize the measure over time, Hurst decided to create a dimensionless ratio by dividing the range by the standard deviation of the observations hence analysis is called "rescaled range analysis". Hurst tired to distinguish a random series from a nonrandom series even the random series is non-Gaussian means not normally distributed. Hurst found that most natural phenomena, including river discharges, temperatures, rainfall and sunspots follow a biased random walk-a trend with noise. The strength of the trend and the level of noise could be measured by how the rescaled range scales with time.

The primary use of R/S analysis is to detect long-term dependence. The presence of long memory components in stock returns has important implications for many of the paradigm of financial economics. If stock returns display long-term dependence, then they exhibit significant autocorrelation between observations widely separated over time. Since the series realizations are not independent over time, realizations from the remote past can help predict future returns, hence giving rise to the possibility of consistent speculative profits. This is in contrast to the martingale or random walk type behavior that many theoretical financial asset pricing models usually assume. Therefore, optimum portfolio decisions may become sensitive to the investment horizon. The presence of long memory in asset returns contradicts the weak form of market efficiency hypothesis, which states that conditioning on past returns, future asset returns are unpredictable. A finding of long memory in asset returns calls into question linear modeling and invites the development of nonlinear pricing models at the theoretical level to account for long memory behavior. Mandelbrot (1971) observes that in the presence of long memory, the arrival of new market information can not be fully arbitraged away. Peters also mentioned that most people wait for confirming information and do not react until a trend is clearly established. Consequently, there will be an uneven assimilation of information. This will cause the stock price movement to follow a biased random walk, instead of random walk.

Whether a time series follows a random walk or not can be detected by the rescaled range analysis or R/S analysis. The R/S analysis is an ideal statistical tool for analyzing the occurrence of rare events and is robust to possible nonlinear process that normality assumption may not be needed. The result of the R/S analysis is the *Hurst Exponent ("H")*, which is a measure of the bias or trend in a time series.

H has broad applicability to all time series analysis, because it is remarkably robust. Hurst's study of time series of natural phenomena may be extended into capital market time series to see whether these series are also biased random walks.

To estimate the Hurst exponent new values are obtained by using the method that described below:

Firstly, begin with a time series, $x = x_1, \ldots\ldots, x_n$. As an example, for capital markets it can be the daily changes in price of a stock index. The mean value, x_m, of the time series x is defined as:

$$x_m = (x_1 + \ldots\ldots + x_n) / n$$

The standard deviation, s_n, is estimated as:

$S_n = n^{-1/2} * \sqrt{} (x_r\text{-}x_m)^2$

The rescaled range was calculated by first rescaling or “normalizing” the data by subtracting the sample mean:

$Z_r = (x_r – x_m); r = 1,\ldots.., n$

The resulting series, Z, has a mean of zero. The next step created a cumulative time series Y:

$Y_1 = (Z_1 + Z_r); r = 2, \ldots\ldots, n$

By definition, the last value of Y (Y_n) will always be zero because Z has a mean of zero. The adjusted range, R_n, is the maximum minus minimum value of the Y_r:

$R_n = \max (Y_1, \ldots\ldots, Y_n) –\min(Y_1, \ldots.., Y_n)$

The subscript, n, for R_n signifies that this is the adjusted range for $x_1, \ldots., x_n$. Because Y has been adjusted to a mean of zero, the maximum value of Y will always be greater than or equal to zero, and the minimum will always be less than or equal to zero. Hence, the adjusted range, R_n, will always be nonnegative.

The Hurst R_n value is divided by the standard deviation of the time series to compare the time series with different characteristics in the same scale.

$(R/S)_n = (\max (Y_1, \ldots\ldots, Y_n) –\min(Y_1, \ldots.., Y_n)) / S_n$

The relation of the value R/S and Hurst exponent is expressed with the equation below:

$(R/S)_n = c* n^H$

R/S : rescaled range

n : number of observations

c : constant

H : Hurst exponent

The Hurst exponent can be approximated by plotting the $\log(R/S_n)$ versus the $\log(n)$ and solving for the slope through an ordinary least squares regression such a following equation:

$$\log (R/S_n) = \log (c) + H^* \log (n)$$

The H value can be interpreted as following:

H=0.50: denotes a random walk and statistically independent (uncorrelated) series. The present does not influence the future. Its probability density function is normal. Such process increases with the square root of time. That is;

For normal distributions variance is stable and finite. So, if the time series were random, the range would increase with the square root of time ($T^{1/2}$ rule) which can be shown as:

In normal distribution, the variance of 5 day returns= $5 * \sigma^2_{\text{daily returns}}$

When we take the square root of both sides of the equation

$$\sqrt{\sigma^2_{\text{daily returns}}} = \sqrt{5 * \sigma^2_{\text{daily returns}}}$$

$$\sigma = \sigma \sqrt{5} = \sigma * 5^{1/2} \quad (1/2=0.50)$$

Where

σ^2 =variance of daily return

σ = standard deviation of daily return

$0.50 < H \leq 1.00$: denotes a “persistent”, or trend-reinforcing series. That is, the data contains long-term memory and has a tendency to follow the current trend in the next period. This process is said to be mean averting. For example; if the series has been up (down) in the past period, then it is more likely to be up (down) in the next period. The strength of the persistence increases as H approaches to 1. Such persistence series are said to be fractional Brownian motion (biased random walks) in terms of nonlinear dynamics, the series displays sensitive to initial conditions.

$0 \geq H < 0.50$: denotes an "anti-persistent" series. That is, the data has a tendency to reverse the current trend. This process is said to be mean-reverting. For example, if the system has been up (down) in the previous period, it is more likely to be down (up) in the next period. The strength of the anti-persistence depends on how close H is to zero.

4.2.2 Brock-Dechert-Scheinkman (BDS) Test

One of the most general and widely used to test for detecting nonlinear dependencies in a time series is the BDS test, which was developed by Brock, Dechert, and Scheinkman in 1986. The BDS statistic has its origins in the correlation dimension plots of Grassberger and Procaccia in 1983, which were developed for studying low-dimensional chaos in time series in physics applications. Brock, Dechert and Scheinkman propose this non-parametric tool as a test of the null hypothesis of an independent and identically distributed ("i.i.d.") time series, with power against virtually all linear and nonlinear, stochastic and deterministic chaotic alternatives, since it is sensitive to any kind of clustering in the phase space. On the other hand, BDS testing may help identify the existence of non-linear dependence found, but not its type.

While estimation of the BDS statistic is non-parametric, the test statistic asymptotically follows a normal distribution with zero mean and unit variance and therefore lends itself for easy hypothesis testing. Moreover, in principle no distributional assumptions need to make about the data under the null hypothesis other than that it is i.i.d. It can be interpreted as a test for non-linearity, if appropriately used in conjunction with ARMA modeling. In a first step, the best fitting ARMA (p,q) is determined and fitted to data, thus eliminating all linearity from the data. Only in a second step is the test applied by running it on the residuals of that ARMA model, which by default must be linearly independent, so that any dependence found in the residuals must be nonlinear in nature.

The BDS statistic, a variant on the correlation dimension, basically measures the statistical significance of the correlation dimension calculations. According to the BDS test, the correlation integrals should be normally distributed if the system under study is independent. The correlation integral was introduced by Grasberger and Proccia in 1963 as a method of measuring the fractal dimension of deterministic data. It is a measure of the frequency with which temporal patterns are repeated in the data. The correlation integral is the probability that any two points are within a certain distance, *e*, from one another apart in phase space. As *e* is increased, the probability scales according to the fractal dimension of the phase space.

There are four steps to calculate Correlation Dimension (Hsieh, 1991, p.7):

1- Remove autocorrelation, if present.

2- Organize the data into n-histories, $(X_t)^n = (X_{t-n+1}, \ldots\ldots, X_t)$, where n is known as the "embedding dimension".

3- Calculate the correlation integrals with the following formula:

$$C_m (e) = (1/N^2)^* \sum_{i,j=1}^{T} Z (e- |X_i - X_j|),\ i \neq j \qquad \text{(eq.1)}$$

where $z(x)=1$ if $e- |X_i - X_j|>0$; 0 otherwise

T = the number of observations
e = distance
C_m = correlation integral for dimension m

The function, Z, counts the number of points within a distance, *e*, of one another. According to theory, the C_m should increase at the rate e^D, with D the correlation dimension of the phase space, which is closely related to the fractal dimension. The correlation integral, from equation (1), calculates the probability that two points that are part of two different trajectories in phase space are *e* units apart. If the X_i in the time series X (with T observations) are independent and X_i series are lagged into “n histories”, then the correlation integral $C_N(e,T)$ is calculated as:

$$C_N(e,T) \rightarrow C_1(e)^N$$

The correlation integral simply fills the space of whatever dimension it is placed in. Brock et al. showed that $|C_N(e,T) - C_1(e,T)^N| * \sqrt{T}$ is normally distributed with a mean of 0. The BDS statistic, w, that follows is also normally distributed:

$$w_N(e,T) = |C_N(e,T) - C_1(e,T)^N| * \sqrt{T} / s_N(e,T)$$

where $s_N(e,T)$ = the standard deviation of the correlation integrals

The null hypothesis under the BDS test is that the increments of the time series are independent and identically distributed. A rejection of the null hypothesis for the BDS test could mean any one or a combination of three major possibilities. These are; one, there are linear serial dependencies in the data; two, the time series is non-stationary; and three, there are nonlinear serial dependencies in the data, either chaotic or stochastic.

Since BDS test is a two-tailed test, we should reject the null hypothesis if the BDS test statistic (w) is greater than the positive critical z-value or less than the negative critical z-value. For example, if $\alpha = 0.05$, the critical z-value = $\pm$ 1.96.

Running the BDS test is far from straightforward, since the test is extremely computationally intensive and special algorithm are needed to make implementation viable. MATLAB software is used to implement the BDS test in this thesis.

4.2.3 Nychka-Ellner-Gallant-McCaffrey (NEGM) Test

Most empirical results on the evidence of nonlinearity and chaos were based on BDS test and a one early method of dominant Lyapunov exponent which is proposed by Wolf et al. (1985, cited in Peters, 1991). However, these methods require long data series and are sensitive to dynamic noise. Recently, Nychka, Ellner, Gallant and McCaffrey (NEGM) (1992) proposed a regression method for the positivity of the dominant Lyapunov Exponent ("LE"). This method has at least two distinguishing features relative to other methods; it is practical, since it allows for the presence of dynamic noise in estimation with a moderate sample sizes.

LE is a most important tool for diagnosing the presence of the sensitive dependence of the initial conditions ("SDIC"). LE measures the average exponential divergence or convergence between trajectories that have "infinitesimally small" differences in their initial conditions (Hilborn, 2000). As it is mentioned before, one of the distinctive features of a chaotic dynamic system is the sensitive dependence on initial conditions. SDIC implies that nearby trajectories become exponentially separated in finite time under the action of a flow. Therefore, the evolution of such a system becomes very complex and essentially unpredictable, except in the short run. For example, if a system is in a state with uncertainty, N_0, at time t_0, then its future state, N, will show a growth of uncertainty determined by;

$$N=N_0 e^{ht}$$

where h is called as Lyapunov Exponent (LE) of the system. A positive value of h implies exponential growth in uncertainty and hence represents an indicator of chaos. Therefore, LE is a most important tool for detecting the presence of SDIC (Peters, 1994).

These explanations about LE can be stated more formally as follows (Hilborn, 2000):

Firstly, an attractor point x_0 and a neighboring attractor point $x_0+\varepsilon$ is considered. Then, each value iterated by iterated map function n times. The absolute value of the difference between those results is presented with the formula below:

$$d_n = | f^{(n)}(x_0+\varepsilon)- f^{(n)} (x_0)|$$

If the behavior is chaotic, this distance is expected to grow exponentially with n, so it can be written as,

$$\frac{d_n}{\varepsilon} = \frac{| f^{(n)}(x_0+\varepsilon)- f^{(n)} (x_0)|}{\varepsilon} = e^{\lambda n}$$

$$\lambda = \frac{1}{n} \ln \left(\frac{| f^{(n)}(x_0+\varepsilon)- f^{(n)} (x_0)|}{\varepsilon}\right)$$

This last pair of equations defines the Lyapunov exponent λ for the trajectory. When the behavior of the system is compared for two similar initial conditions, λ is related to the rate at which the subsequent trajectories diverge. A bounded system with $\lambda>0$ is one operational definition of chaotic behavior.

5. EVIDENCE ON THE EXISTENCE OF NONLINEARITY AND CHAOTIC BEHAVIOR ON ISTANBUL STOCK EXCHANGE

More recently, the published empirical literature has concentrated on testing economic and financial time series for the presence of nonlinear dependencies using various measures indicative of complex dynamics.

Abhyankar *et al* (1995, p.865) have explained the critical importance of these issues in finance as follows:

1- It helps to arrive at a conclusion about market efficiency in stock exchange markets. For example, the presence of a well-behaved nonlinear structure would be inconsistent with market efficiency.

2- A chaotic process is defined as one characterized which is sensitive dependence on initial conditions. The complexity of the process may make it impossible for agents to identify market behavior, even though researchers can uncover the true model. More importantly, however, sensitive dependence on initial conditions means that knowing the function driving the market price may be insufficient to guarantee a profit, because forecast accuracy may degenerate too rapidly to leave time for profitable trades to be executed.

The purpose of the thesis is to investigate the evidences on existence of nonlinear dynamics and chaos in ISE by using composite index data. There are number of tests for nonlinearity and chaos in the literature. In this thesis, three well known tests are implemented respectively, Rescaled Range Analysis, BDS test and NEGM Lyapunov exponent test, to reach a conclusion about three questions presented below:

1- Is there long-term memory in ISE composite index data?

To answer this question R/S analysis has been implemented to data by estimating Hurst exponent.

2- Is there nonlinear dependence in ISE composite index data?

Existence of nonlinear dynamics in ISE is tested with BDS test under the null hypothesis of the increments of the time series are independent and identically distributed.

3- And if nonlinear dependence exists, is nonlinear structure characterized by low dimensional chaos?

The sensitive dependence of initial conditions of ISE composite index data has been measured by estimating Lyapunov exponent.

5.1. DATA AVAILABLE FOR STUDY

The data used in this study is daily closing price of Istanbul Stock Exchange which is obtained from the Central Bank of Turkey. The data consists of daily closing index of a value weighted index comprised of the one hundred traded stocks called National-100 Index. The sample period spans 03/01/1989 to 19/05/2006. According to the data some observations are shown as zero, these observations are dropped and 4321 observation remained.

Before the nonlinearity tests, giving the descriptive statistics should be helpful to understand the structure of the data. Some of the descriptive statistics for the 4320 observation of daily return of ISE closing index are shown in the Table 5.1 below. The daily returns of the ISE composite index are calculated as the change in logarithm of closing stock market indices of successive days:

$$\mathbf{r_t = ln\ (ISE_t / ISE_{t-1})} \qquad t = 1,2, \ldots, n-1$$

where,

ISE_t = index at the day t

r_t= return at the day t

n= the number of observations

Peters (1991) suggested using logarithmic data since taking the first difference may not only ensure that the time series are stationary but also it is a common practice in standard econometric work to “whiten” a time series.

When Table 5.1 is examined the minimum, maximum, mean, median, standard deviation, skewness coefficient and kurtosis values can be seen. Last two lines of the table show the estimated third and fourth central moments for the data. Skewness coefficient is -0.0712730 that is negative that the data slightly skewed to the left. Kurtosis value is 6,0565000 which is greater than three implies a leptokurtic distribution means that fat tails is a distribution property for the data.

Table 5.1

Summary Descriptive Statistics: Daily ISE Composite Index Returns

Minimum	-0.1997851
Maximum	0.1777358
Mean	0.0021437
Median	0.0019589
Standard Deviation	0.0300610
Skewness	-0.0712730
Kurtosis	6.0565000

Table 5.2 shows the percentile values of the raw data. One percent of the data is less than -0.08331, where the minimum is -0.1997851. Ninety-nine percent of the raw data is less than 0.08461, where the maximum value is 0.1777358. Approximately, forty percent of the data is negative which means that sixty out of hundred times ISE composite index has increased with compare to previous day.

Table 5.2

Percentiles of Daily ISE Composite Index Returns

Percentile	Values of Percentile
1	-0,08331
10	-0,03064
20	-0,01806
30	-0,01051
40	-0,00385
50	0,00196
60	0,00789
70	0,01443
80	0,02266
90	0,03658
95	0,05038
99	0,08461

Quartile is a measure of dispersion statistic and divides the series into parts such that for example first quartile value indicates that twenty five percent of the observation is less than this number. Table 5.3 shows the quartile values of the raw data. First twenty five percent of the raw data is less than -0.0140899 and last twenty percent of the data is greater than 0.01807355.

Table 5.3

Quartile Values for Daily ISE Composite Index Returns

Quartiles		
25^{th}	50^{th}	75^{th}
-0,0140899	0,00195889	0,01807355

Frequency Table of the raw data and Histogram frequency table of return series are also constructed in Table 5.4 and Figure 5.1, respectively. Class width has decided to be equal 0.00581, but since some classes become empty and some frequency was less than six, these classes has decided to add in other classes. As a result of this, width of the first and last classes is not equal to the 0.00581. Table 5.4 shows the frequency, relative frequency and cumulative frequency values. When the eighteenth class is examined: there are 462 observations and it is ten percent of the data, and fifty three percent of the data is less then the upper bound 0.00350.

Table 5.4

Frequency Table

Classes		Frequency	Relative Frequency	Cum.Rel. Frequency
-0,19979	-0,09524	22	0,0051	0,0051
-0,09524	-0,08943	11	0,0025	0,0076
-0,08943	-0,08362	9	0,0021	0,0097
-0,08362	-0,07782	13	0,0030	0,0127
-0,07782	-0,07201	14	0,0032	0,0160
-0,07201	-0,06620	12	0,0028	0,0188
-0,06620	-0,06039	33	0,0076	0,0264
-0,06039	-0,05458	28	0,0065	0,0329
-0,05458	-0,04878	40	0,0093	0,0421
-0,04878	-0,04297	58	0,0134	0,0556
-0,04297	-0,03716	70	0,0162	0,0718
-0,03716	-0,03135	112	0,0259	0,0977
-0,03135	-0,02554	137	0,0317	0,1294
-0,02554	-0,01974	223	0,0516	0,1810
-0,01974	-0,01393	309	0,0715	0,2525
-0,01393	-0,00812	345	0,0799	0,3324
-0,00812	-0,00231	399	0,0924	0,4248
-0,00231	0,00350	462	0,1069	0,5317
0,00350	0,00930	410	0,0949	0,6266
0,00930	0,01511	355	0,0822	0,7088
0,01511	0,02092	327	0,0757	0,7845
0,02092	0,02673	221	0,0512	0,8356
0,02673	0,03254	171	0,0396	0,8752
0,03254	0,03834	144	0,0333	0,9086
0,03834	0,04415	109	0,0252	0,9338
0,04415	0,04996	67	0,0155	0,9493
0,04996	0,05577	59	0,0137	0,9630
0,05577	0,06158	37	0,0086	0,9715

0,06158	0,06738	27	0,0062	0,9778
0,06738	0,07319	13	0,0030	0,9808
0,07319	0,07900	28	0,0065	0,9873
0,07900	0,08481	13	0,0030	0,9903
0,08481	0,09062	11	0,0025	0,9928
0,09062	0,09642	10	0,0023	0,9951
0,09642	0,10223	21	0,0049	1,0000
TOTAL		**4320**		

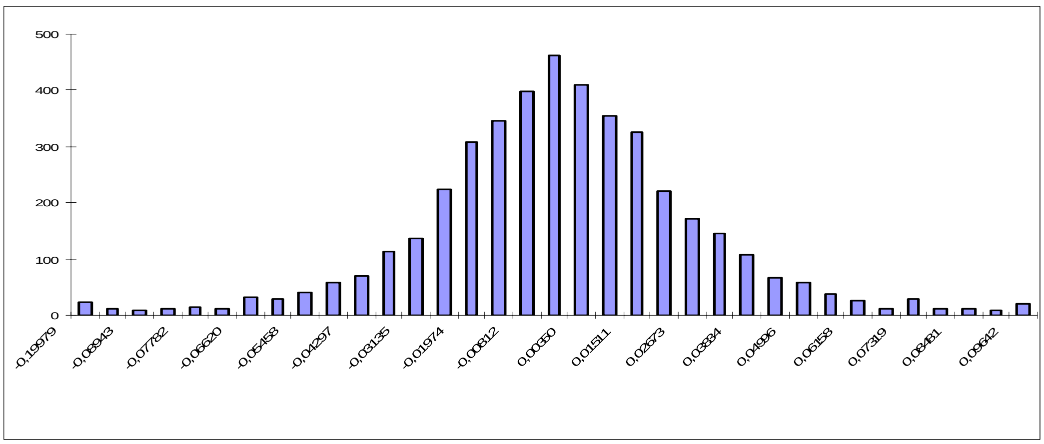


Figure 5.1: Histogram of Frequency Table for Daily ISE Composite Index Returns

Since both descriptive statistics (skewness and kurtosis) indicate deviations from normal values, it can be said that the observed distribution are not normally distributed. Therefore, the Jarque-Bera statistic is calculated. In this case the result of the test indicates a rejection of the null hypothesis, which is normality of the time series. Finally, simple unit root test is performed in order to test the stationarity of the selected series. Augmented Dickey-Fuller (ADF) test is used. The result for the ADF test statistic is compared with MacKinnon critical values for rejection of hypothesis of a unit root at 1% significant level. In this case the time series seem to be stationarity- at least in the form, which is incorporated in the ADF test. The summary of the test results can be seen in Table 5.5 presented below:

Table 5.5

Jargue-Bera and ADF Test Results

Jargue-Bera	1682.1
Probability (p-value)	0,000000
ADF test statistic	-10.5691
1% critical value (*)	-3,9655
(*)MacKinnon critical values for rejection of hypothesis of a unit root	

5.2. FINDINGS OF THE ANALYSIS APPLIED ON THE EXISTENCE OF NONLINEARITY AND CHAOTIC BEHAVIOUR IN ISTANBUL STOCK EXCHANGE

5.2.1 Rescaled Range (R/S) Analysis

To test the existence of long-term memory in a series, it is subject to rescaled range (R/S) analysis. Since R/S analysis is robust enough to detect long-term dependence if the distribution of the series is not normal, R/S analysis is an appropriate tool to detect long-term dependence in the data used in this study. Tables 5.6 and 5.7 present the results of the R/S analysis of the ISE Composite Index daily returns and Figure 5.2 shows the log(n)/log(R/S) plot.

Table 5.7 shows that the *H* exponent for daily ISE Composite Index return series is 0.6009. The high R-square (99.46%) and low standard error of estimate (0.0344) illustrate the goodness of fit of the regression model for estimation. The value of H is greater than 0.5 implies that persistence exists in series. Today's data affect all future data. If the prices have been up during the current period, there is a probability of 60.09% that they are likely to be up during subsequent period. Such kind of persistent trend is said to be biased random process, or fractional Brownian motion. The time series is persistent implies that the investors' interpretation of events is not reflected in the price immediately. The interpretation manifests itself and becomes a bias in return, which is different from that suggested by Efficient Market Hypothesis (Chu, 2003, p.212).

Table 5.6

R/S Analysis Results: Daily ISE Composite Index Returns

ISE			
N	*Log(n)*	*R/S*	*log(R/S)*
5	0,6990	1,7500	0,2430
6	0,7782	2,0326	0,3081
7	0,8451	2,2790	0,3577
10	1,0000	2,9786	0,4740
20	1,3010	4,6757	0,6698
25	1,3979	5,4142	0,7335
26	1,4150	5,5818	0,7468
40	1,6021	7,3151	0,8642
50	1,6990	8,7182	0,9404
52	1,7160	8,9698	0,9528
65	1,8129	10,2813	1,0120
80	1,9031	11,9400	1,0770
100	2,0000	13,1169	1,1178
104	2,0170	13,1493	1,1189
130	2,1139	15,3144	1,1851
200	2,3010	19,7220	1,2950
208	2,3181	20,7664	1,3174
260	2,4150	23,1121	1,3638
325	2,5119	23,8939	1,3783
400	2,6021	25,6402	1,4089
520	2,7160	32,6726	1,5142
650	2,8129	39,1803	1,5931
1040	3,0170	49,1492	1,6915
2159	3,3343	68,1767	1,8336
2160	3,3345	68,4428	1,8353

Table 5.7

Regression Results: Daily ISE Composite Index

	ISE
Hurst Exponent	0,6009
P-value of the regression coefficient	0,0000
Constant	-0,1125
R^2	0,9946
Standard Error of Estimate	0,0344

Note: Dependent variable: log(R/S)
Independent variable: log(n)

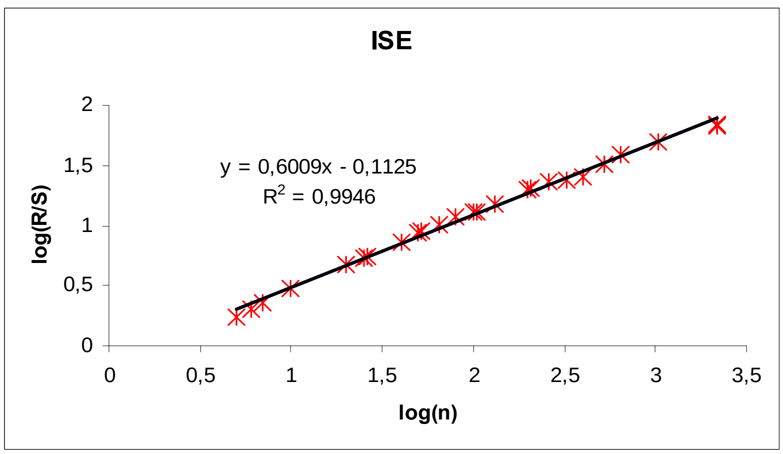


Figure 5.2 The log(R/S) versus log(n) Plot

5.2.2. Test of nonlinearity: BDS Test

To test the presence of non-linearity, the logarithmic index return series in ISE is subject to BDS test. Firstly, the linear dependence in the original time series has removed from the system by pre-whitening the returns series employing appropriate autoregressive model and capturing the residuals. These captured residuals now become the whitened returns series subject to the statistical analysis for BDS test. The order of the autoregressive model used were determined by relying on the minimum Akaike Information Criterion estimate as ARMA (4,5).

The hypotheses of the BDS test are:

H_0: The error terms of data in time series is independent and identically distributed (i.i.d.).

H_1: The error terms of data in time series are not i.i.d. (this implies that time series is non-linearly dependent, since linear dependence has been removed)

The BDS statistics are computed for the underlying dimension of m= 2, 3, 4 and 5 and for **ε** values of **ε** = 0.50, 1, 1.5, 2. A level of significance (α) of 5% is taken and thus the critical value for the test is ±1.96, means that the null hypothesis should be rejected if the BDS test statistic is greater than 1.96 or less than -1.96. Table 5.8 indicated that all test statistics of error terms are greater than the critical value of 1.96 significantly under different embedding dimension and ratio of tolerance to standard deviation. Thus, the null hypothesis of i.i.d. for data should be rejected. The results strongly suggest that both series are non-linearly dependent at the 5% level of significance.

Table 5.8

BDS Test Results: Daily ISE Composite Index Returns

ε/σ	**Embedding Dimension (*m*)**	**BDS Test Statistics**
2	2	18,2029
2	3	21,8764
2	4	23,2040
2	5	24,3055
1,5	2	18,0127
1,5	3	22,6371
1,5	4	24,9910
1,5	5	27,1437
1	2	17,1545
1	3	22,7903
1	4	26,5936
1	5	30,4078

0,5	2	15,7321
0,5	3	22,0438
0,5	4	27,3037
0,5	5	33,5553

5.2.3. Test of Chaos: NEGM Lyapunov Exponent

A method of testing for the chaos is to compute the dominant Lyapunov exponent. Chaos is defined by the sensitive dependence on initial values or the existence of a positive Lyapunov exponent.

The available methods of analyzing experimental or observational data for evidence of chaos are based on calculating a few key points that characterize the dynamics, in particular fractal dimensions and Lyapunov exponents. These methods give reliable results if the data are abundant (thousands or tens of thousands of values), if measurement error is near 0, and if the data really come from a deterministic system. With limited data or a system subject to nonnegligible stochastic perturbations, the results may be incorrect or ambiguous (McCaffrey, Ellner, Gallant, &Nychka, 1992, p.682).

With the help of the NEGM test the dominant lyapunov exponent of a dynamical system can be extracted from a scalar time series even it is short and noisy. Nychka et al. has proposed a regression method for the positivity of the dominant LE of a nonlinear system based on "Jacobian Method". In this study, Jacobian matrix is estimated by using the neural network model to estimate function by non-linear least squares to select optimum parameter of L,m,k. The parameter triple (L,m,k)[*] is selected as L=1, m=3 and k=3 with the respective optimized value of the BIC criterion.

The hypotheses of the NEGM test are;

H_0= Lyapunov exponent is negative or Time series is not chaotic or $\lambda \leq 0$

H_1= Lypunov exponent is positive or Time Series is chaotic or $\lambda \geq 0$

[*] Numbers in parentheses represent the BIC selection of the parameter triple, (L,m,k), where L is the time delay parameter, m is the number of lags in the autoregression and k is the number of units in the hidden layer of the neural net.

In this thesis, only feed-forward single hidden layer networks with a single output considered. Neural network is implemented by using the neural network toolbox in MATLAB software package. The Lyapunov exponent is also computed using the MATLAB software package with a Lyapunov MATLAB code.

The estimated Lyapunov exponent for the time series is -0.6457867. Since Lyapunov exponent point estimate is negative, than zero null hypothesis can not be rejected, means that time series is not chaotic.

6. CONCLUSION

This study has given a list of some recent works in finance about the existence of nonlinear dependence, chaotic behaviour and long memory in stock exchange markets. Most of the empirical research, as summarized in Literature Review Section, provides proofs for the existence of the nonlinear dependence or chaotic behaviour in financial markets.

Since, research using conventional econometric methods has uncovered several deviations from market efficiency in the behaviour of stock prices, several tests capable of detecting nonlinear patterns as well as linear patterns in the data have been developed with the efforts of statisticians, econometricians and physicists.

One of the contributions of this research has been simply to provide additional information about the Istanbul Stock Exchange which is the primary stock market of a developing economy, Turkey.

In this study, the behavior of ISE Composite Index Return series has been examined using the rescaled range (R/S) analysis, the BDS test and NEGM Lyapunov Exponent estimation.

The Hurst exponent found in the rescaled range analysis is greater than 0.5 reveal that the series is persistent, fractal and self-similar with long-term memory. However, the value of the Hurst exponent is relatively low (around 0.6), which indicate that series has strong noisy component.

Nonlinearity was found in the time series by the BDS test. The test statistics are very significant which indicate a strong evidence of nonlinearity. Nonlinearity is one of the indications of chaotic behaviour.

To test for chaos a recently introduced test, NEGM test on the Lyapunov Exponent, is used. The result of the test shows that dominant Lyapunov exponent is negative means that the time series is not chaotic.

The findings of nonlinearity and long-term memory provide some implications to financial analysts as follows:

- The distribution of the time series is leptokurtic. This finding reveals the possibility of leptokurtic distribution rather than normal distribution. In fact, the practice of assuming normal distribution is not because of fitting the data but because of enabling us to apply the classical statistical tools.

- The presence of memory effect in time series revealed by the rescaled range analysis indicates that time series are not generated by a pure random walk model but a biased random walk process.

- The nonlinearity found in the time series implies that why most of the traditional econometric tools fail to model time series.

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APPENDIX

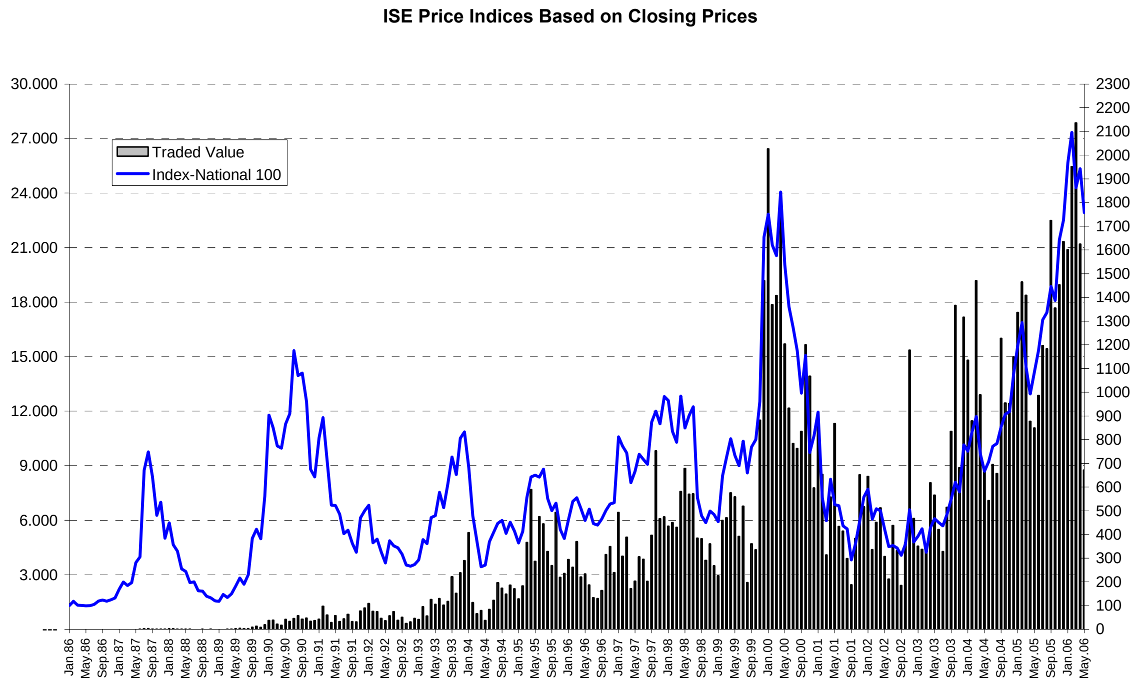

Figure A.1 ISE Price Indices Based on Closing Prices

ISTANBUL STOCK EXCHANGE: 5 YEAR STATISTICAL COMPARISON

	2002	**2003**	**2004**	**2005**	**YTD 2006**
Number of Companies	288	285	297	304	306
Market Capitalization (US$*)	34401.5	69002.8	980.730.453	1.628.144.058	1.771.800.657
Total Vol.-Stocks (US$*)	70756.3	100165.4	1.477.550.939	2.016.918.705	739.720.307
Total Vol.-Stocks (# Shares*)	33933250.9	59099780.4	696.146.314.987	81.083.376	266.213.818
Index	368.2	778.4	1075.12	1726.23	1862.36

* in millions Source:ISE

Table A.1 5 Year Statistical Comparison of ISE